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ON THE AMPLIFYING POWER OF OBJECTIVES AND OCULARS IN THE COMPOUND MICROSCOPE.

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A great deal has been said and written on this subject and still the matter is not as clear and accurate as could be desired.

The European opticians usually name their objectives and oculars in an arbitrary manner, as No. I, No. II, No. III, etc., or A, B, C, etc., but these designations give no clue to the amplifying powers, except that the lower numbers or earlier letters usually indicate the lower magnifying powers. The No. I objective of one maker does not, however, necessarily correspond with the No. I of another maker, and the No. I objective does not necessarily correspond in amplifying power with the No. I ocular of the same maker.

The English makers have attempted to avoid this confusion and, to introduce a degree of uniformity, have long adopted a system of nomenclature based upon the amplifying power; that is, if a combination of lenses magnifies equal to a single convex lens of one-inch focus, the combination is *called* a one-inch; if the same as a single lens of one-quarter inch focus, it is *called* a quarter-inch, etc., etc. This system has long been in use in England and this country for objectives, and more recently has been extended to oculars (or Eye Pieces, as they are commonly called). This was supposed to give a very simple and accurate means for determining the power of any objective or ocular or combination of objective and ocular, provided only that they were correctly named by the maker and were used on a tube of the standard length. The rule commonly in use is based upon the assumption of the arbitrary distance of ten inches as the distance of distinct vision, and that the number of times the focal length is contained in ten inches is the amplifying power; so that a *one*-inch lens would magnify ten times, a one-fourth inch forty times, a one-tenth inch one hundred times, etc., etc. The image of the object projected by the objective being again magnified by the ocular, it was further assumed that the same rule would apply and therefore that the amplification produced by the combination of a $\frac{1}{10}$ objective with a one-inch ocular would be found by multiplying the assumed power of the $\frac{1}{10}$ objective (100) by the assumed power of the one-inch ocular (10)=1000 diameters. And so, by the application of this simple rule, every owner or user of objectives and oculars of the new nomenclature could calculate correctly the theoretical power of each and of any combination, with the understanding that, if the distance between the optical center of the objective and that of the ocular varied, the amplifying power would vary in proportion.

The object of the present paper is, 1st to show the incorrectness of this rule, in that the real image projected by a simple convex ten inches from its optical center is *not* amplified the number of times the focal length is contained in ten inches and that the same rule of amplification that is true and correct for the objective that projects a *real* image cannot be true and correct for the ocular, which projects a *virtual* image; and 2nd, to present a correct method of determining the (linear) amplifying power of any objective or ocular, correctly named on the equivalent focal length system, and of any combination of such objective and ocular at any given distance between their optical centers.

Now, in so far as the amplication of the image projected by the objective is concerned, the distance of distinct vision is of no consequence whatever, but the result is governed solely by the well known optical law that, "The linear dimensions of object and image are directly as their distances from the optical center of the lens." The correctness of this can be demonstrated by actual measurement, for the image is as real as the object, and its distance from the optical center of the lens and its dimensions can as easily be measured.

Before proceeding to the further discussion of this subject it may be well to define some of the optical terms which I shall be obliged to use. I am quite aware that, for the majority of my hearers, this is a work of supererogation that almost savors of impertinence; but there are always beginners amongst us and it is for their sakes that I insert these elementary definitions.

Definitions.

Optical Center:—The point through which all rays traversing a lens with parallel directions at incidence and emergence must pass. In double convex or double concave lenses it lies in the interior of the lens; in plano-convex or plano-concave lenses it lies on the curved surface; while in a meniscus of either kind it lies outside the lens altogether.

Principal Axis:—The straight line passing through the centers of curvature of both faces of a double convex, a double concave, or a meniscus lens, or passing through the center of curvature of the curved face and cutting at right angles the plane face of a plano-convex or a plano-concave lens is called the Principal Axis; the optical center is always in this line.

Secondary Axes:—All other straight lines passing through the optical center are called "Secondary Axes."

Principal Focus:—The point at which rays originally parallel to the principal axis are made to converge (approximately) to one point.

Focal Length:—The distance from the Optical Center to the Principal Focus.

Conjugate Foci:—Rays emerging from a point more distant than the principal focus on one side of a convex lens and passing through the lens will be brought to a focus at a point on the other side of the lens and the points thus related are called Conjugate Foci.

As one conjugate focus advances from infinite distance (parallel rays) to the principal focus, the other recedes from the principal focus to infinite distance, the most distant focus always moving most rapidly, and the least distance between them is therefore attained when they are equidistant from the optical center, in which case the distance of each from the optical center is 2f. and their distance from each other 4f. If either is less than the principal focus then the other becomes negative; that is, the rays are no longer brought to a focus on the *opposite* side of the lens, but are only rendered less divergent, as if coming from a more distant point on the *same* side, and this point from which they *appear* to come (the more distant of the two) is called the Virtual Conjugate Focus. In this case, as one conjugate focus advances .towards the optical center, the other advances in the *same* direction till they become coincident.

Secondary principal and conjugate foci exist in each of the secondary axes of a convex lens and are under the same laws as the primary foci.

Each point in an object has its conjugate point in the image of it formed by a lens, and this image, if on the opposite side of the lens, is real and inverted; if on the same side, is virtual and erect. The linear dimensions of the object and image are directly as their distances from the optical center of the lens; so that, if the object be nearer than the image, then the image is magnified, and *vice versa.*

Formulae.

The formulae for the determination of the conjugate foci when:

f = Principal focus (or focal length)
p = One conjugate focus
p' = the other conjugate focus.

When the conjugate foci are on opposite sides of the lens (real image):

$$\frac{1}{p} + \frac{1}{p'} = \frac{1}{f}$$

This formula suffices for the determination of either of the conjugate foci, the other conjugate focus and the focal length being given; or of the focal length, the two conjugate foci being given; and as it applies equally well to points in the secondary axes, it suffices equally to determine the distances of the object and image (and thence their relative linear dimensions), if one of these distances and the focal length of the lens be given.

When the conjugate foci are on the same side of the lens (virtual image) the formula becomes :

$$\frac{1}{p} - \frac{1}{p'} = \frac{1}{f}$$

The plus sign here becomes minus, or, to express it in other terms, as the two conjugate foci are now on the same side of the lens it is the difference instead of the sum of their reciprocals that equals the reciprocal of the focal length. This formula is as applicable as that for real conjugate foci to the determination of the places, and therefore of the relative linear dimensions, of image and object; but, of course, the change of sign produces marked differences in the results when the given quantities are the same; that is to say, with a given focal length and image distance, the distance of the object, and therefore the ratio between its linear dimensions and those of the image, will vary according as the image is real or virtual.

In the compound microscope we have to deal with both real and virtual images; the real produced by the objective and the virtual by the ocular.

The real and inverted image produced by the objective becomes in its turn the object of which the ocular produces a virtual image, erect so far as it is concerned, but, of course, still inverted as regards the original object.

The degree of amplification of the real image produced by the objective depends upon two factors; 1st, the focal length of the objective, and 2d, the distance from its optical center at which the image is formed. It can be formed at any distance from the focal length of the objective up to infinity.

In most microscopes of the English and American model, the tube is of such length that the image is formed at a distance of about ten inches, and that distance is therefore taken as the basis of calculation and the formula then is

$$\frac{1}{p}\ \frac{1}{\text{(object distance)}} + \frac{1}{p'}\ \frac{1}{\text{(image distance)}} = \frac{1}{f}\ \frac{1}{\text{(focal length)}};$$

or, substituting the image distance 10 for p′:

$$\frac{1}{p} + \frac{1}{10} = \frac{1}{f}$$

With this formula, let us work out the case of a 5-inch objective; then

$$\frac{1}{p} + \frac{1}{10} = \frac{1}{5}$$

$$\frac{1}{p} = \frac{1}{5} - \frac{1}{10} = \frac{1}{10}$$

$$\frac{p}{1} = \frac{10}{1}$$

$$p = 10;$$

that is, the object and image are equidistant from the optical center and therefore of equal size, and there is no amplification. Of course, it can be assumed that the image distance p′ is greater than 10 inches, as in case the draw tube is used, when the formula will show that, with a 5-inch objective, the image is larger than the object, or, p′ can be taken as less than 10 inches, when the formula will show that, with a 5-inch objective, the image is less than the object.

Keeping 10 inches for our image distance, let us take the case of a $\frac{1}{5}$ inch objective, then $f = \frac{1}{5}$

$$\frac{1}{p} + \frac{1}{10} = \frac{1}{\frac{1}{5}}$$

$$\frac{1}{p} = \frac{1}{\frac{1}{5}} - \frac{1}{10} = \frac{50}{10} - \frac{1}{10} = \frac{49}{10}$$

$$p = \frac{10}{49}$$

$$\text{a (amplification)} = \frac{10}{\frac{10}{49}} = 49 \text{ times.}$$

By this formula we can easily calculate the amplification of the real image projected at 10 inches by any simple-convex lens, if the focal length of the lens be known; and I present herewith a table so calculated for most of the focal lengths used for microscope objectives. (Table A.)

It will be noted that the amplications obtained are, in every case, less than those obtained by the number of times the focal length is contained in 10 inches, and the reason is that one conjugate focus (the image distance) being at less than infinite distance the other conjugate focus (the object distance) *must* be at a greater distance than the focal length, and therefore a quantity greater than the focal length must be used for the divisor, and the quotient (the amplification) must be less.

As a 1-inch simple-convex lens amplifies the image projected by it at 10 inches from its optical center 9 times, a one-inch objective should do the same (without reference to its actual focal length). If it fails to do so, if the image projected by it at 10 inches from its optical center is amplified more or less than 9 times, then the objective has been incorrectly named; it is not a one-inch objective, but something else.

The Ocular,

Having disposed of the real image projected by the objective, we come to the virtual image projected by the ocular; here the formula is $\frac{1}{p} - \frac{1}{p'} = \frac{1}{f}$;
substituting the image distance 10 for p′ we have $\frac{1}{p} - \frac{1}{10} = \frac{1}{f}$.

With this formula, let us work out the case of a 5-inch ocular:

$$\frac{1}{p} - \frac{1}{10} = \frac{1}{5}$$

$$\frac{1}{p} = \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$$

$$p = \frac{10}{3}$$

$$a = \frac{10}{\frac{10}{3}} = \frac{30}{10} = 3 \text{ times}$$

The wide difference of this result from that obtained for a lens of the same focal length used as an objective shows very plainly the absurdity of using, as many of us have done, and as many of the books teach, the same general rule for determining the amplifying power of objective and ocular, viz., to divide ten inches by the focal length expressed in inches.

I present herewith a table of amplifications of virtual images projected at ten inches by simple lenses corresponding to the most commonly used oculars. (Table B.)

The total amplifying power of any combination of objective and ocular is obtained by multiplying the amplifying power of one by that of the other.

For instance the total amplifying power of a micrcscope with tube of standard length carrying a 1-inch objective and a 2-inch ocular should be $9 \times 6 = 54$,
instead of $10 \times 5 = 50$,
as per the usual rule.

It is to be noted, however, that while the formulæ here given are theoretically correct for the objective and ocular respectively and are applicable to any image distances by substituting the desired image distance for p', as 10 was substituted for it in the examples given, yet there are many complications in the practical application of any formula to the determination of the actual amplification obtained by the modern compound microscope; among these complications are

First—The highly complex construction of many objectives, making it very difficult to ascertain with any degree of accuracy, the position of the optical center, which difficulty is still further increased when the objective under consideration is furnished with a correction arrangement for various thicknesses of cover-glass, which, by varying the relative positions of its component lenses, varies its actual and nominal focal length and the position of its optical center. The exact position of the optical center of the ocular is also, at times, difficult of determination.

Second—The refractive condition of the observer's eye is also a factor in the amplification under which the image is finally *seen*, for the reason that the dioptric system of the observing eye becomes, in fact, a part of the ocular, and any difference of its refractive power greater or less that required to focus on the retina rays proceeding from a radiant situated at the given image distance, must be added to or subtracted from the refractive power of the ocular and thus decrease or increase its focal length. That is to say, a person who can and does accommodate for precisely 10 inches while looking through the microscope will, if all the other conditions are rigidly complied with, see the image under the exact amplification indicated by the formula, while one, who by reason of myopia or of excessive use of the muscle of accommodation accommodates for a less distance, will see it under a greater amplification, and the emetrope or hyperope who relaxes his accommodation to less than that required to bring rays from a radiant at 10 inches to a focus on his retina, will see it under a less amplification than that indicated by the formula. For instance, let us take the case of the combination of 1-inch objective and 2-inch ocular for which we have found the total amplification, when image distances are taken as 10 inches in case of both objective and ocular, to be

$$9 \times 6 = 54.$$

If the observing eye be accommodated for just 10 inches, the image will be seen clearly and under an amplification of x 54. If, however, the eye is accommodated for any other distance, then the image will not be clearly seen and a change must be made in the adjustment of the microscope to make it clear. The reason is that the excess or defect of the refraction of the eye above or below what is required to accommodate it for 10 inches has, in effect, been added to or taken from the refractive power of the ocular.

Suppose an observer, as the result of myopia or from spasm of, or voluntary action of the muscle of accommodation, accommodates for a distance of 5 inches instead of 10 inches; he has, in effect, added to the refractive power of the ocular the refractive power of the lens which represents the difference between a refractive power of ten inches and of five inches. The refractive power of a lens is the reciprocal of its focal length. Hence the equation is

$$\frac{1}{5} - \frac{1}{10} = \frac{1}{10}$$

Then refractive power of ocular + Excess of Ref. of Eye = Ref. power of Eye and ocular taken as one, or

$$\frac{1}{2} + \frac{1}{10} = \frac{6}{10} = \frac{1}{1.66}$$

Hence the resulting amplification will be as if the ocular had a focal length of 1.66 instead of 2 and the formula will be

$$\frac{1}{p} - \frac{1}{10} = -\frac{1}{1.66}$$

$$\frac{1}{p} = \frac{1}{1.66} + \frac{1}{10} = \frac{11.66}{16.6}.$$

$$p = \frac{16.6}{11.66}$$

$$a = \frac{10}{\frac{16.6}{11.66}} = \frac{116.6}{16.6} = 7.$$

Hence for the observer whose eye is accommodated for 5 inches the expression for total amplification of the 1-inch objective and the 2-inch ocular will be

$$9 \times 7 = 63.$$

On the other hand let us take a case much more unlikely to occur in practice, that of a person who by reason of excessive hypermetropia or paralysis of accommodation, is unable to focus any but parallel rays upon his retina, or, in other words, to accommodate for any point nearer than infinite distance. If such an observer make use of the same combination of objective, he has in effect, subtracted from the refractive power of the ocular the refractive power of the lens, which represents the difference between a refractive 0 and of 10 inches $= \frac{1}{10}$; then

$$\frac{1}{2} - \frac{1}{10} = \frac{4}{10} = -\frac{1}{2.5}$$

Hence the resulting amplification will be as if the ocular had a focal length of 2.5 instead of 2, and the formula will be

$$\frac{1}{p} - \frac{1}{10} = -\frac{1}{2.5}$$

$$\frac{1}{p} = \frac{10}{25} + \frac{1}{10} = \frac{125}{250} = \frac{1}{2}$$

$$p = 2$$

$$a = \frac{10}{2} = 5$$

Hence for the emetropic observer whose accommodation is entirely relaxed, or for any observer whose eye is accommodated for parallel rays, the total amplification of the 1-inch objective and the 2-inch ocular will be

$$9 \times 5 = 45$$

For strict accuracy, the change in the position of the image produced by the objective, if the adjustment for the different eyes be produced in the usual way by means of the fine or coarse adjustment moving the objective to or from the object, should be taken into consideration, but the amount is so small, about $2\frac{1}{2}$ per cent., in the first case and 5 per cent. in the second, that it may be neglected without seriously impairing the practical accuracy of the general result, while, if the adjustment for different eyes be made with the draw-tube moving the ocular only, the position of the image produced by the objective is not changed, and therefore, so far as it is concerned, the original formula remains strictly correct.

NOTE.—In the Huyghenian Ocular (the form most commonly in use) the Field-lens, while mechanically part of the ocular is optically part of the objective, in that it contributes to the formation, not of the virtual image projected by the ocular, but of the real image projected by the objective, upon which it acts negatively, diminishing its size while increasing the superficial area brought into view at one time. So that, in this form of ocular it is the Eye-lens alone that contributes to the reamplification of the image, but the negative action of the Field-lens must, of course, always be taken into consideration when attempting to determine the amplifying power of a Huyghenian ocular by calculation.

TABLE "A."

Amplification (linear) of real images projected at 10 inches from optical center by simple bi-convex lenses.

Focal length of Lens in inches.	Linear Amplification of Image.	Focal length of Lens in inches.	Linear Amplification of Image.
5.	1.	$\frac{1}{4}$	39.
4.	1.5	$\frac{1}{5}$	49.
3.	2.33	$\frac{1}{6}$	59.
2.	4.	$\frac{1}{7}$	69.
1.	9.	$\frac{1}{8}$	79.
$\frac{3}{4}$	12.33	$\frac{1}{9}$	89.
$\frac{2}{3}$	14.	$\frac{1}{10}$	99.
$\frac{1}{2}$	19.	$\frac{1}{12}$	119.
$\frac{4}{10}$	24.	$\frac{1}{16}$	159.
$\frac{1}{3}$	29.	$\frac{1}{25}$	249.

TABLE "B."

Amplification (linear) of Virtual Images projected at 10 inches from optical center by simple bi-convex lenses.

Focal length of Lens in Inches.	Linear Amplification of Image.	Focal length of Lens in Inches.	Linear Amplification of Image.
5.	3.	$.\frac{3}{4}$	14.
4.	3.5	$\frac{1}{2}$	21.
3.	4.33	$\frac{4}{10}$	26.
2.	6.	$\frac{1}{3}$	31.
$1.\frac{1}{2}$	7.73	$\frac{1}{4}$	41.
1.	11.		